Synthesis of Low-Reflection Waveguide Joint Systems*

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Summary—**Some characteristics of flat-flange type joints are analyzed. Experimental evidence is given proving that it is possible to reproduce, in practice, waveguide joints which have identical complex-reflection coefficients. Such joints can be combined to form large joint systems by means of a synthesis method, which keeps the over-all reflection coefficient to a minimum. Both theory and experimental data are presented.**

I. Introduction

IN high-quality, wide-band, frequency-modulated microwave communication systems the ultimate distortion level is quite often determined by the end and internal reflections of the transmission line.[1-4] In and above the S band (2000 Mc) the transmission loss in a coaxial line becomes high and the required tolerances for a coaxial branching system are extremely tight. It is difficult to obtain a low input reflection coefficient with a coaxial antenna input in a wide frequency band.

Thus it becomes mandatory to use waveguide components in high-quality transmission systems. Such a system assures low resistive loss and low end reflections; however, it introduces new types of reflection sources at the waveguide joints. From communication-system distortion standpoint two parameters characterize these internal reflections; the reflection coefficient of one joint, and the so-called "ripple factor," which is the ratio of the maximum and the average reflection coefficient in the operational frequency band at the input of the waveguide system.

The first parameter depends on the construction of the joint, while the ripple factor is determined by the positions of the joints along the line. By means of a simple synthesis method it is possible to calculate: 1) the allowable number of joints and their position for a given maximum reflection coefficient, and 2) frequency band and total length of transmission line.

II. Characteristics of the Individual Waveguide Joint

A. General Behavior

The complex reflection coefficient of the flat-flange type joint depends on three factors:[5-9]

1) Size difference between two joining waveguides.
2) Misalignment between the two waveguides.
3) Surface geometry and electrical conductivity along the flange surface.

The surface geometry depends on the surface roughness, the connecting bolt pressure, and the elasticity of the flange material close to the mating surface. The electrical conductivity depends on the quality and purity of the material and is also affected by the long-term corrosion processes.

According to the experimental data it is possible by careful manufacturing to obtain a joint which can be represented by approximately the series impedance

$$Z = R(f) + \frac{1}{j2\pi fC} + jX(f), \tag{1}$$

where $R(f)$ represents the frequency-dependent loss resistance along the contact surface, C is the series capacity between the two mating flanges, and $X(f)$ represents the effect of step and misalignment, which can be either inductive or capacitive. However, it can be found, experimentally that

$$\left| R(f) + \frac{1}{j2\pi fC} \right| \gg \left| X(f) \right|, \tag{2}$$

if the manufacturing tolerances are within small, but practically possible, limits. Two important facts follow from (1) and (2):

1) The magnitude of the reflection coefficient can be relatively constant in ± 15 per cent frequency band if the resistive and capacitive components of (1) are comparable.
2) It is possible to obtain approximately identical complex reflection coefficients for each joint along the waveguide system, if the surface quality and flange pressure on the flanges are nearly equal.

The utilization of these facts leads to the application of the synthesis method to obtain an optimally located waveguide joint system.

Table I summarizes the practically possible tolerances and their corresponding theoretical reflection coefficients

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[9] H. A. Wheeler and H. Schwiebert, "Step-twist waveguide components," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-3, pp. 44–51; October, 1955.

for S-band waveguides (4.300×2.150 inch inside dimensions). The sum of these individual reflections gives an indication of the upper limit of the reflection coefficient caused by the size and misalignment tolerances.

With the tolerances indicated in Table I the theoretical maximum value of the reflection coefficient is 0.071 per cent. However, the measured reflection coefficient on such a system is five to ten times larger, depending on the surface roughness, flatness and contact pressure on the flanges. The difference can be obviously interpreted as the effect of the gap between the flanges, and the order of this difference shows that the approximation indicated in (2) is reasonable.

Fig. 1 shows three typical measured flange surface qualities: Curve A represents the measured roughness of a cast brass flange; B shows the surface roughess after precision milling; C after lapping. From these samples, it is clear that the actual conducting flange surface can be several times larger than the nominal surface. This results in a larger resistive loss.

Besides the loss, RF energy is stored in the microscopic voids between the flanges. This energy storage can be represented by an equivalent series-capacitive reactance.

The effective conductivity between two flanges depends on the material and the surface roughness. No detectable difference can be measured in the reflection coefficient of silverplated or aluminum flanges maintaining the same surface roughness. However, large deterioration in the conductivity considerably affects the reflection coefficient. For example, when the surface quality was achieved by means of a lapping process, it was found that the reflection coefficient was considerably higher than the expected value. After the highly resistive lapping compound was partly removed, the reflection coefficient was improved slightly in spite of the fact that the chemical cleaning process resulted in a somewhat rougher surface.

From the assumed equivalent series impedance of the joint (1) it can be seen that, as the frequency increases, the capacitive reactance decreases more rapidly than the series resistance increases. (The first term in (1) increases approximately with the square root of frequency, while the second term decreases inversely with the frequency.) Therefore the reflection coefficient should decrease slightly with the frequency. The experimental data proved this assumption.

In practice, the gap size can be reduced by

1) Decreasing the surface roughness.
2) Adding a thin, soft metallic layer on top of the finished mating surfaces.
3) Applying a large uniform pressure on the contact area.

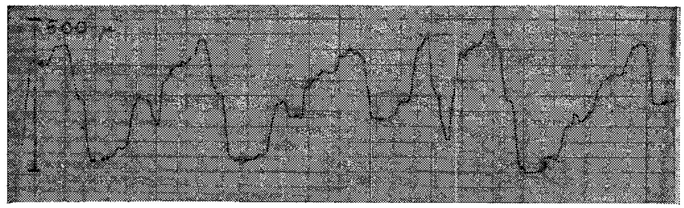
B. Methods Reducing the "Gap Effect"

Improvement in the Surface Roughness: Tables II and III show some measured reflection coefficient values

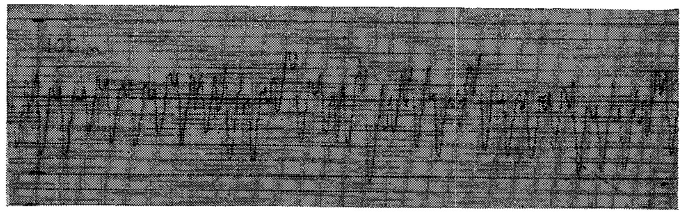
TABLE I

EFFECTS OF GEOMETRICAL IMPERFECTIONS

Type of Error	Tolerance	Reflection Coefficient (per cent)
Step	±0.0015 inch	0.050
Transversal Misalignment	±0.0005 inch	0.010
Polar Misalignment	±0.1°	0.001
Total		0.071



(a)



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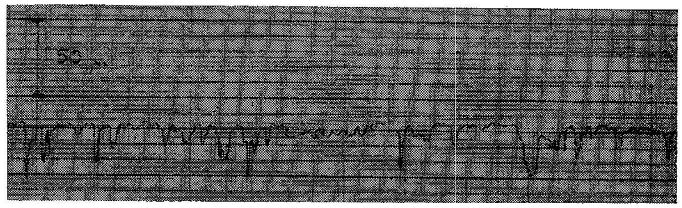


(c)

Fig. 1—Typical flange surface qualities.

for different flange surface qualities in the 1800–2300-Mc frequency band. The first of these tables offers simple proof of the gap effect. The input reflection coefficients were measured on a waveguide run which consisted of 10 sections, each 20 feet long. (The "average of the maxima" is defined in the following way: The 500-Mc-wide operational frequency band is divided into 10 50-Mc-wide sub-bands. The largest value of a reflection coefficient is taken from each of the sub-bands and then the 10 values are averaged.) The effect of the size and misalignment tolerances was negligible, relative to the effect of the flange surface. A peak-to-peak surface roughness of 100 μ inch was measured on the surfaces.

In arrangement A, $\frac{1}{8}$-inch-long spacers were placed between the two flanges, while in arrangement B these spacers were eliminated. Physically the lossy, reactive air-gap was reduced by a factor of 2 in the second arrangement. It can be seen that both the peak and the average reflections were reduced in the second arrangement approximately by a factor of two.

Table III shows the reflection coefficients of wave-

TABLE II
EFFECT OF VARYING THE NUMBER OF AIRGAPS

Arrangement	Maximum Input Reflection Coefficient (per cent)	Average of the Maxima in the Input Reflection Coefficient (per cent)	Average of the Maxima per Joint (per cent)
A. Waveguide Run with Elementary Spacers in the Joints	18.2	11.6	1.05
B. Waveguide Run without Elementary Spacers	9.2	6.2	0.56

TABLE III
EFFECT OF SURFACE ROUGHNESS

Surface Roughness in μ inch			Average of the Maxima in the Input Reflection Coefficient per Joint (per cent)	Note
Case	Peak to Peak	rms		
1	500	62	0.49	Flanges Cut from Extruded Material
2	100	12	0.38	The same as Case 1 with Standard Milling
3	60	8	0.33	The Same as Case 1 with Precision Milling
4	60	8	0.30	The Same as Case 3 with Gold Interconnecting Layer

TABLE IV
THE EFFECT OF THE QUARTER-WAVE SPACER

Condition	Maximum Input Reflection Coefficient (per cent)	Average Input Reflection Coefficient (per cent)	Average Reflection Coefficient per Joint (per cent)	Average of the Maximum Reflection Coefficient per Joint (per cent)
5 Sections without Spacers, 500-μ Inch Peak-to-Peak Surface Roughness.	6.5	2.0	0.33	0.81
5 Sections with Spacers, 500-μ Inch Peak-to-Peak Surface Roughness.	4.0	1.19	0.20	0.50

guide joints which have different surface roughness. These data were measured with quarter-wave spacers between the waveguide flanges. Table III also shows that the reflection coefficient varies considerably with the surface roughness. However there is a practical limit (probably 8 μ inch rms) beyond which there is no economical return.

Application of a Soft Interconnecting Layer: The idea of using some soft interconnecting metallic layer (indium, gold, etc.) between the two mating flanges is one solution to decrease the air-gap in the joint, or to ease the surface roughness requirement for the same electrical quality. The utilization of gold plating was tried in case 4 in Table III where the reflection coefficient decreased by an additional 10 per cent.

Optimum Contact Pressure on the Flanges: It is obvious from the construction of a flange that the air-gap between two flanges has a minimum value which is a function of the connecting bolt tension. At low tension the gap is not closed completely, while at high tension levels the flange warps and the air gap increases again.

Ten $\frac{1}{4}$-inch-diameter standard stainless-steel bolts were used to maintain the connecting pressure on the flanges. The reflection coefficient was recorded as a function of the torque applied on the individual bolts. The effect of pressure was investigated in the 5–30 foot-pound torque range and it was found that the optimum value was between 10–15 foot-pounds.

Quarter-Wave Spacer Compensated Joint: The previous results indicated that under certain conditions the reflection of a joint is limited by the gap effect, which may be approximately equal for each joint. If the spacing between two identical joints is a quarter wavelength, then their net reflection is zero. In a frequency band the compensation can be only partial, but as Fig. 2 shows, the improvement is theoretically more than 6 db in the useful band of the RG-104 waveguide. Fig. 3(a) shows a photograph of such a quarter-wave spacer and Fig. 3(b) shows the reflection coefficient of a 5-section waveguide run with and without quarter wave spacers. During this test relatively rough flange surfaces were used with a 500 μ inch surface finish. Table IV summarizes the conclusions which can be drawn from the two curves on Fig. 3(b). It indicates that the quarter wave spacer resulted in a 4.6-db improvement in the average reflection coefficient, which is quite close to the theoretically estimated value.

III. DETERMINATION OF A LOW-REFLECTION JOINT SYSTEM ALONG A WAVEGUIDE RUN

A. *Uniformly Distributed Joints*

Fig. 4 shows an L-long waveguide run which is divided by n joints. If the higher-order internal reflections are negligible, then the input reflection coefficient

$$\Gamma_{in} = \Gamma_0 + \Gamma_1 e^{j2\beta_g l_{01}} + \cdots + \Gamma_{n-1} e^{j\beta_g l_{0,n-1}}, \quad (3)$$

where β_g is the propagation constant of the guide and l_{0k} is the length between the first and kth joint. When

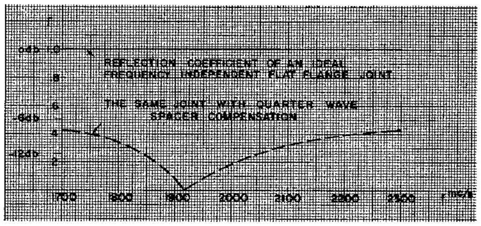
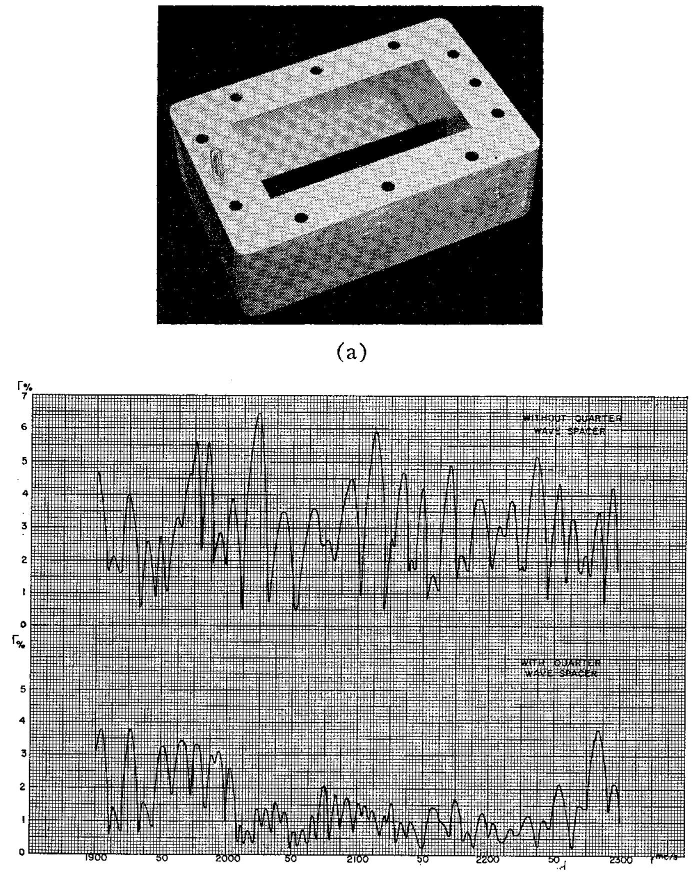


Fig. 2—Theoretical reflection coefficient of a quarter-wave spacer compensated joint.



(a)

(b)

Fig. 3—(a) Quarter-wave waveguide spacer. (b) Reflection coefficient of a five-section waveguide run with and without quarter-wave spacer.

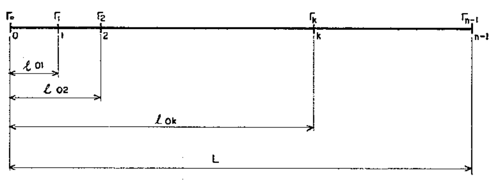


Fig. 4—Geometrical arrangement of a joint system.

all the individual joint reflections are equal in magnitude and phase ($\Gamma_k = \Gamma_i = \Gamma$) and the sections are equal in length, then the normalized input reflection coefficient

$$\Gamma_N = \frac{\Gamma_{in}}{\Gamma} = e^{j(n-1)\frac{w}{2}} \frac{\sin \frac{nw}{2}}{\sin \frac{w}{2}}, \tag{4}$$

and

$$|\Gamma_N| = \left| \frac{\sin \frac{nw}{2}}{\sin \frac{w}{2}} \right|, \tag{5}$$

where

$$w = 2\beta_g l = 4\pi \frac{l}{\lambda_g} \cdot$$

Some important characteristics of this simple function are:

1) $|\Gamma_N|$ is periodic as a function of w,

$$w = 4\pi l \frac{1}{\lambda_g} = \frac{4\pi}{c}\left[1 - \left(\frac{c}{2af}\right)^2\right]^{1/2} clf. \tag{6}$$

where c is the velocity of light.

In a narrow frequency band the factor under the square root is approximately constant, *i.e.*, $|\Gamma_N|$ is approximately periodic with frequency.

2) The mth maximum of $|\Gamma_N|$ occurs at a point where

$$2\pi \frac{l}{\lambda_{gm}} = m\pi, \quad \text{and here} \quad |\Gamma_N|_M = n. \tag{7}$$

The ratio of two adjacent resonant wavelengths is

$$\frac{\lambda_{gm+1}}{\lambda_{gm}} = \frac{m}{m+1} \cdot \tag{8}$$

For example if $l = 20$ feet $= 609.6$ cm and $f_0 = 1997$ Mc, then $\lambda_{gm} = 20.68$ cm and $m = 59$. The difference between two adjacent resonant frequencies is approximately 1.7 per cent.

3) From a practical standpoint not only $|\Gamma_N|_M$ is interesting, but also the average of $|\Gamma_N|$ throughout the operational frequency band. This average can be defined as

$$\Gamma_{av} = \frac{1}{w_2 - w_1} \int_{w_1}^{w_2} |\Gamma_N| \, dw, \tag{9}$$

where w_1 and w_2 correspond to the two limits of the frequency band. Furthermore, the input reflection coefficient can also be characterized by

$$R = \frac{|\Gamma_N|_M}{\Gamma_{av}} \cdot \tag{10}$$

R will be called the ripple factor in this paper. Fig. 5 shows the variation of the ripple factor and $|\Gamma_N|_M$ as a function of the number of joints. It can be seen that both R and $|\Gamma_N|$ increase rapidly with the number of joints if the waveguide sections are equal in length.

B. Synthesis of a Smoother Input Reflection Coefficient

A simple synthesis method is now given which results in a joint distribution with a low $|\Gamma_N|_M$ and a low ripple factor. It is assumed that the absolute value of the input reflection coefficient $|\Gamma_{in}|$ can be represented by the following function:

$$F_n = |\Gamma_{in}|_n = \left| \prod_{k=1}^{n/2} (1 + e^{j(\lambda_{g0}/\lambda_g)} e^{jkw}) \right|$$

$$\left[2^n \prod_{1}^{n/2} (1 - \cos kw) \right]^{1/2} = \left[2^n \prod_{1}^{n/2} G(k) \right]^{1/2} \quad \text{if } \lambda_g = \lambda_{g0}$$

$$= \left[2^n \prod_{1}^{n/2} (G(k) + \pi\Delta \sin kw) \right]^{1/2} \quad \text{if } \lambda_g = (1 + \Delta)\lambda_{g0}. \quad (11)$$

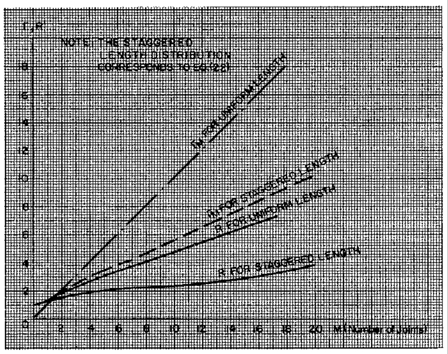


Fig. 5—The maximum reflection coefficient and ripple factor of the uniform and staggered joint distribution vs number of joints.

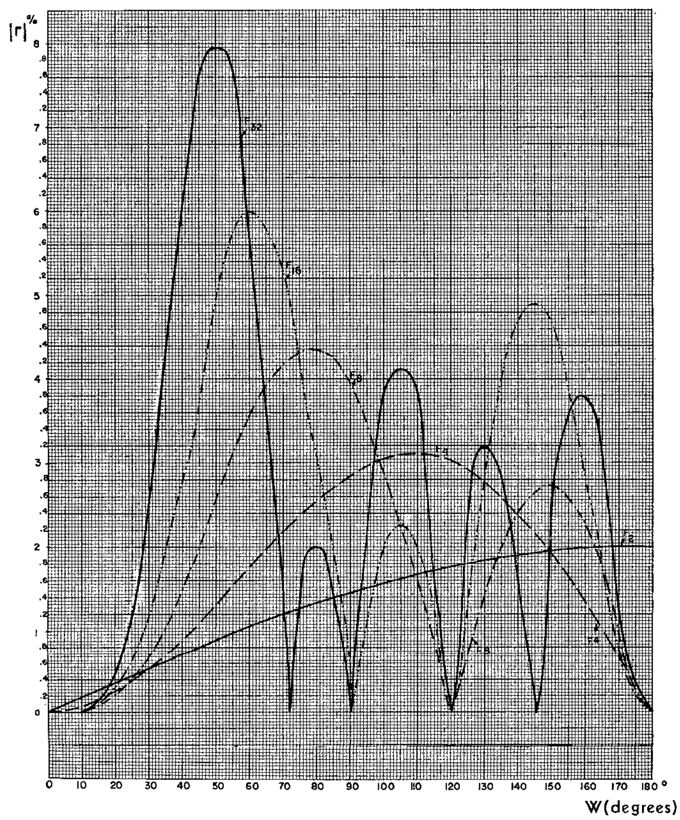
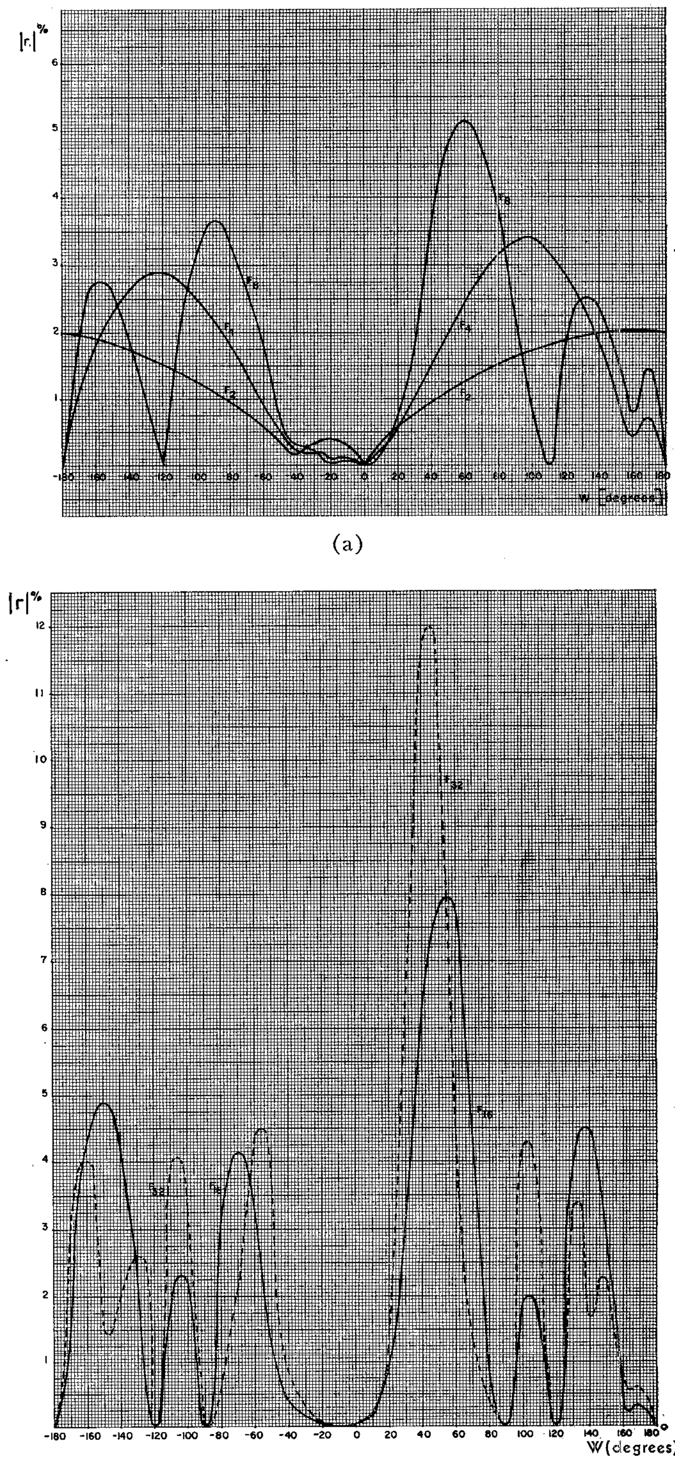


Fig. 6—The F_n functions at the center of the frequency band.

The F_n functions are given in Figs. 6 and 7 for various n as a function of w in the center and at the end of a ± 12.5 per cent frequency band. As can be seen from the curves, the F_n functions have very desirable characteristics as input reflection coefficients, *viz.*, they increase relatively slowly as n increases.

If (11) is selected as the input reflection coefficient of the waveguide system, then it is easy to generate the



(b)

Fig. 7—The F_n functions at the limits of a ± 12.5 per cent frequency band.

corresponding joint distribution. For this purpose it is sufficient to analyze the form of (11) in the center of the frequency band ($\lambda_g = \lambda_{g0}$). The reflection coefficient for the A_2 joint distribution in Fig. 8 is

$$|\Gamma_2| = |1 + e^{j2\beta_g(4l-\lambda_{g0}/4)}| = |1 - e^{j4w}| = \sqrt{2}[1 - \cos 4w]^{\frac{1}{2}}. \quad (12)$$

In this distribution a $4l - \lambda_{g0}/4$ separation is assumed between the two joints. If an identical second joint system will be added to the first in such a way that its position will be shifted relative to the first by $3l - \lambda_{g0}/4$ toward the end of the line, then the resultant input reflection coefficient

$$|\Gamma_4| = \left| |\Gamma_2| + |\Gamma_2| e^{j2\beta_g(3l-\lambda_{g0}/4)} \right| = |\Gamma_2| \, |1 - e^{j3w}| = 2[(1 - \cos 4w)(1 - \cos 3w)]^{1/2}. \quad (13)$$

This is the A_4 joint distribution in Fig. 8. Repeating the process similarly, the end result is

$$|\Gamma_{16}| = 4[(1 - \cos w)(1 - \cos 2w)(1 - \cos 3w) \cdot (1 - \cos 4w)]^{1/2}, \quad (14)$$

which is (11) in the center of the band for $n = 8$. Fig. 8 shows the whole process graphically. In the end results (A_{16} joint distribution) the lines are almost equal, but simple joints as well as quarter-wave spacer separated joints should be employed. There are practical cases when from manufacturing standpoint it is not desirable to use different line lengths, or larger than one unit reflection at one joint. In such cases it is possible to obtain an approximation of the theoretical requirements by the use of quarter-wave spacers. For instance, a good equivalent of the A_{16} distribution is A_{16}' and A_{16}'' (see Fig. 9). In Table V some measured data are shown. From it a comparison can be made between the system with uniform length and spacers, and the A_{16}' distribution. The sensitivity of the synthetized reflection coefficient to the length tolerances of the individual sections was measured on three manufactured A_{16}'-type waveguide run. Table VI indicates the manner in which the input reflection coefficient varied as a function of the rms line length variations. According to the table the maximum value of the reflection coefficient shows more tolerance sensitivity than the average value.

IV. Method of Measurement

There are two type of difficulties in the measurement technique of low-reflection waveguide joints or joint systems.[10–12]

[10] W. A. Rawlinson, J. Hooper, and R. Branch, "A Swept-Frequency Method of Locating Faults in Waveguide Aerial Feeders," Post Office Res. Sta., Dollis Hill, London, N.W. 2, Res. Rept. No. 20209; June, 1959.
[11] A. C. MacPherson and D. M. Kerns, "A new technique for the measurement of microwave standing-wave ratios," Proc. IRE, vol. 44, (Correspondence), p. 1024; August, 1956.
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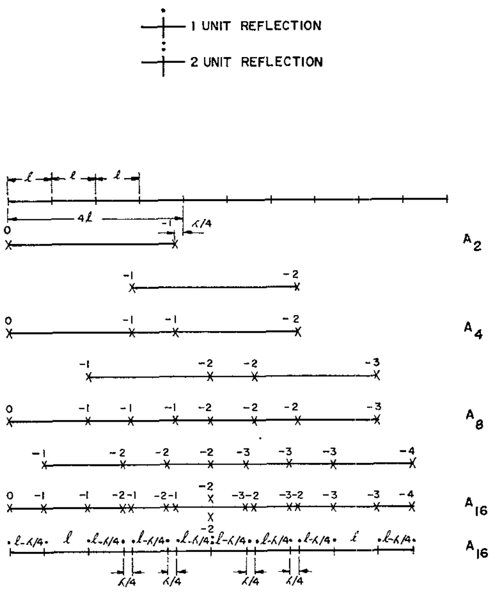

Fig. 8—Determination of the joint distribution, which corresponds to the F_{16} function.

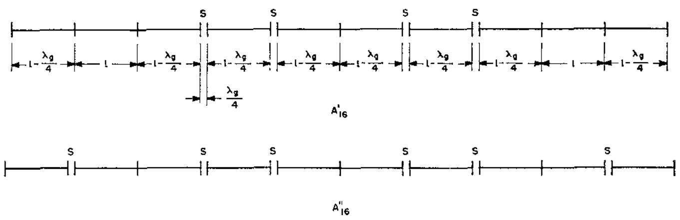

Fig. 9—Two approximate equivalents of the A_{16} distribution. S = quarter-wave spacer. All lengths are $1' = 1 - \lambda g/4$.

TABLE V

Comparison Between the Uniform Length and the A_{16}' Distribution

Distribution	Number of Joints	Input Reflection Coefficient Theoretical*		Input Reflection Coefficient Experimental	
		Γ_M per cent	Γ_{av} per cent	Γ_M per cent	Γ_{av} per cent
Uniform Length	11	4.94	0.99	5.07	1.2
A_{16}'	11	2.70	0.77	2.50	0.76

* 0.45 per cent reflection coefficient was assumed for one joint and for calculation purpose, the actual A_{16}' was replaced by A_{16}.

TABLE VI

Effect of the Length Tolerances

RMS Deviation from the Nominal Length (in inches)	Γ_M per cent	Γ_{av} per cent
0.041	4.56	1.12
0.033	3.57	1.15
0.016	2.50	0.76

1) If the reflection coefficient of one waveguide joint is in the order of less than 1 per cent, it is not convenient to measure accurately such a low-reflection coefficient with the slotted line technique.
2) It is relatively easy to use a sweep method and a high directivity directional coupler if the reflection coefficient is larger than 1 per cent, *i.e.*, in the case of joint systems. This method, however, is very time consuming because every change in the physical arrangement must be carried out along the full waveguide run.

For time saving, some of the preliminary measurements were carried out on systems which contained few joints. For this purpose an intermediate-power (lw) sweep generator and high-quality waveguide directional coupler were developed (7-db coupling, more then 50-db directivity). The high-power level and high directivity extended the absolute measuring limit down to reflection coefficients of 0.5 per cent, while a 0.05 per cent relative change was still detectable.

The effects of the various parameters (steps at the joints, surface finish, flange pressure, etc.) were studied on a relative basis, while the final numerical data were recorded using point-by-point measurements on larger joint systems.

Fig. 10 shows the test set-up which was used for the individual-joint reflection study, as well as the various length modulation experiments. Fig. 11 shows a typical scope presentation of the reflection coefficient. The line under test was the A_{16}' distribution. The peak value on the scope is equivalent to a 2.5 per cent reflection coefficient (3.5 unit). Within the represented frequency range (1780–2300 Mc), the total output power variation of the sweep generator and the coupling variation of the coupler was less than 1 db/100 Mc.

From an applicational standpoint the most important datum is the value of the peak and average reflection coefficient as a function of the line length (joints). Fig. 12 shows the theoretical maximum and average reflection coefficients and the points indicate some measured data. The calculations and measured data correspond to quarter-wave spacer compensated joints, and the length modulation is based on (11). It can be seen that there is significant agreement between the theoretical and experimental results.

V. Conclusions

With precise manufacturing processes, it is possible to produce waveguide joints which have identical electrical characteristics. Such joints can be very useful building blocks in a system with specially located flanges. In the resulting network both the maximum and the average input reflection coefficients may have unusually low values.

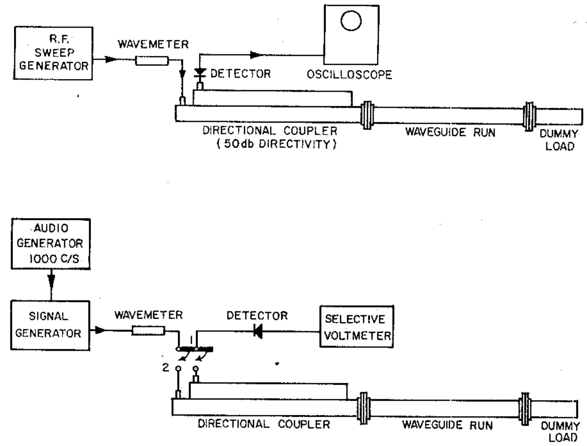

Fig. 10—Test setup for the reflection coefficient measurement.

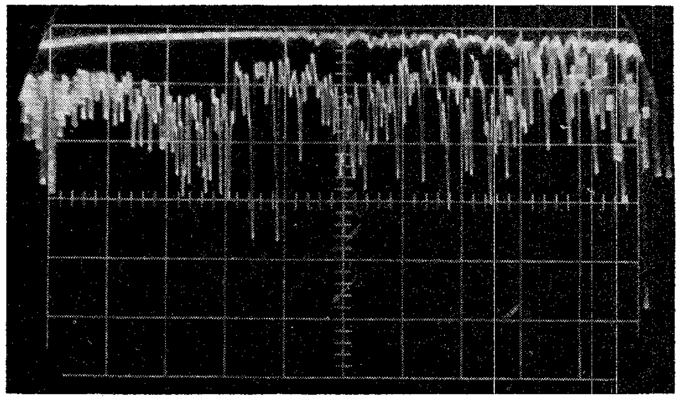

Fig. 11—Scope presentation of the reflection coefficient for the A_{16} distribution.
$F=1780$–2300 Mc.
$\Gamma=2.5$ per cent.

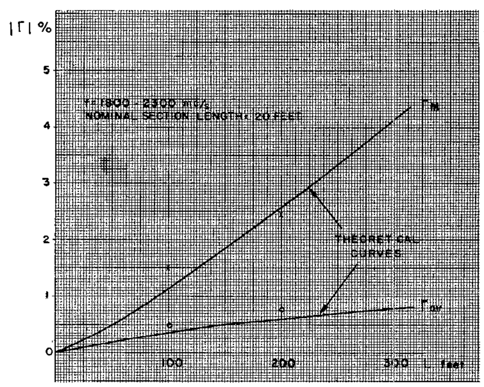

Fig. 12—Reflection coefficient of straight waveguide run vs length. Joints are quarter-wave spacer compensated. Length modulation corresponds to (11).

VI. Acknowledgment

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